Ch 3 notes

Monday, November 9, 2020 9:26 AM

Unit 3 Notes Finance

Assignments for Pre-College Math
Chapter 3: Consumer Math and Financial Management

Day	**Assignment** (Due the next class meeting)
11/9 11/12	3.1 Notes and HW
11/10 11/13	3.2 notes and HW
11/16 11/18	3.3notes and hw
11/17 11/19	3.4 notes and hw
11/20 11/24	3.5 notes and hw
11/23 11/30	
12/1 12/3	
12/2 12/4	Ch 3 review and Practice Test
12/7 12/9	Ch 3 Test
12/8 12/10	
12/11 C day	
12/14-12/17	FINALS

Pre-College Math **Chapter 3 Guided Notes: Consumer Mathematics**

Section 3.1: Simple Interest

Example 1: Express $\frac{5}{8}$ as a percent.

$8\overline{)5.000}$ $.625$ 62.5%

Example 2: Express 0.47 as a percent.

47%

Example 3: Express each percent as a decimal:

a. 19%   b. 0.25%

$.19$ $.0025$

1

Simple Interest Principle (starting amt)

amt interest **I = Prt** - time (years)
rate (decimal)

To calculate simple interest:

(The rate *r*, is expressed as a <u>decimal</u> when calculating simple interest.)

Example 1: A student took out a simple interest loan for $1800 for two years at a rate of 8% to purchase a new car. Find the interest of the loan.

$P = 1800$

$t = 2$

$r = 8\% \rightarrow .08$

$A = 1800(.08)(2)$

$= \$288$

Example 2: Fred made an investment for 5 years at a rate of 6%, and ending up earning $120 in interest. How much was the investment for?

$I = 120$

$t = 5$

$r = .06$

$120 = P(.06)(5)$

$\frac{120}{.3} = \frac{P(\cancel{.3})}{\cancel{.3}}$

$P = \$400$

Compounding Interest

Total amount; Principle (starting amt); rate (decimal); time (years); # times compounded

$$\boxed{A = P(1 + \frac{r}{n})^{nt}}$$

Example 1: You deposit $2000 in a savings account at Hometown Bank, which has a rate of 6%, compounded annually.

a. Find the amount, *A*, of money in the account after 3 years.

$P = 2000$

$r = .06$

$n = 1$

$t = 3$

$A = 2000(1 + \frac{.06}{1})^{1(3)}$

$= 2000(1 + .06)^3$

$= 2000(1 + .06)^3$

$= 2000(1.191016)$

$= \$2382.03$

x^4

b. Find the interest.

$$\begin{array}{r} 2382.03 \\ -\ 2000 \\ \hline \$\ 382.03 \end{array}$$

Example 2: $5000 deposited into an account with an interest rate of 7%, compounded annually.

a. How much money would be in the account after 4 years?

P = $5000
r = .07
n = 1
t = 4

A = 5000
A = 50
A =
7^4

b. How much interest was paid over 4 years?

3.2 Compound Interest

.53
.54
.53

■**Compound Interest:** To calculate the compound interest paid **more** than once a year we use

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

Final amount

principle (starting amt)

time (years)

rate (decimal)

times compounded

Note: We will assume 365 days per year and round answers to the nearest cent (__2__ decimal places.)

Example 3: You deposit $7500 in a savings account that has a rate of 6%. The interest is compounded monthly.

a. How much money will you have after five years?

P = 7500
r = .06
n = 12
t = 5

$A = 7500\left(1 + \frac{.06}{12}\right)^{12(5)}$

$= 7500\left(1 + \frac{.06}{12}\right)^{60}$

$10,116.38

b. Find the interest after five years.

10 116.38
− 7500

$2,616.38

Example 4: How much money would be in an account earning 4.2% interest, compounded quarterly, if $3000 is deposited and left in the account for 10 years?

P = 3000
r = .042
t = 10
n = 4

$A = 3000\left(1 + \frac{.042}{4}\right)^{40}$

$4555.90

Example 5: An investment is made into a fund that earns 4.2%, compounded daily. If $8,000 is initially invested, how much money will be in the account after 5 years? How much interest will have accrued?

P = 8000
n = 365
t = 5
r = .042

$8000\left(1 + \frac{.042}{365}\right)^{365(5)}$

$9 869.31

interest
$1869.31

■Compound Interest: Continuous Compounding

1. For n compounding periods per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$

2. For **continuous** compounding: $A = Pe^{rt}$

Example 6: You decide to invest \$8000 for 6 years and you have a choice between two accounts. The first pays 7% per year, compounded monthly. The second pays 6.85% per year, compounded continuously. Which is the better investment?

$P = 8000$ $\quad A = 8000\left(1+\frac{.07}{12}\right)^{12(6)}$

$t = 6$ $\quad \$12160.84$

$r = .07$

$n = 12$

$P = 8000$

$t = 6$

$r = .0685$

$A = 8000e^{(.0685)(6)}$

$8000e^{.411}$

Example 7: Charlie invests \$3000 in an account that earns 5% interest, compounded continuously. How much money would be in the account after 10 years?

$A = Pe^{rt}$

\$4946.16

3.3 Annuities

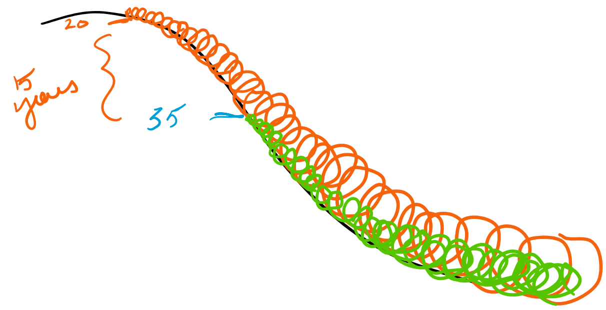
■Annuities: An ***annuity*** is a sequence of equal payments made at equal time periods.

The ***value of an annuity*** is the sum of all payments plus all interest paid.

■Annuity Interest Compounded Once a Year

If P is the deposit made at the end of each compounding period for an annuity that pays an annual interest rate r (in decimal form) compounded n times per year, the value, A, of the annuity after t years is:

$$A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}$$



Example 1: To save for retirement, you decide to deposit $1000 into an IRA(individual retirement account) at the end of each year for the next 30 years. If you can count on an interest rate of 10% per year compounded annually.

a. How much will you have from the IRA after 30 years?

$$A = 1000 \frac{\boxed{\left(1+\frac{.1}{1}\right)^{30} - 1}}{\left(\frac{.1}{1}\right)} = \$164{,}494.02$$

b. Find the interest earned.

$1000(30) = 30000$

$$\begin{array}{r} 164494.02 \\ -\ 30000 \\ \hline \$134494.02 \end{array}$$

Interest

Example 2: At age 25, to save for retirement, you decide to deposit $200 into an IRA at the end of each month at an interest rate of 7.5% per year compounded monthly.

a. How much will you have from the IRA when you retire at age 65?

$$A = 200 \left[\left(1 + \frac{.0}{2}\right)^{480} \ \middle/ \left(\frac{.075}{12}\right)\right] = \$604764.43$$

b. Find the interest earned.

$200(12)(40)$

$\$96000$

$$\begin{array}{r} 604764.43 \\ -\ 96000 \\ \hline 508{,}764.43 \end{array}$$

The interest is more than ___5___ times the amount of your contributions to the IRA.

~5.5

Example 3: At age 30, to save for retirement, you decide to deposit $1000 into an annuity each quarter at an interest rate of 5.5% per year compounded quarterly.

a. How much will you have from the annuity when you retire at age 60?

$t = 30$
$n = 4$
$r = .055$
$P = 1000$

$$1000 \frac{\left[\left(1+\frac{.055}{4}\right)^{120} - 1\right]}{\left(\frac{.055}{4}\right)} = \$301{,}729.22$$

b. Find the interest earned.

$1000(4)(30)$

The interest is more than ____________ times the amount of your contributions to the IRA.

Example 4: At age 20, to save for retirement, you decide to deposit \$3000 semi-annually into an annuity at an interest rate of 4.5% per year compounded semi-annually.

a. How much will you have from the annuity when you retire at age 60?

b. Find the interest earned.

Section 3.4: Installment Buying

■**Fixed Installment Loans**

- The ***amount financed*** is what the consumer ____________:

Amount financed = cash price – down payment.

- The ***total installment price*** is the _______ of all monthly payments plus the down payment:
-

Total Installment Price = Total of all monthly payments + down payment.

- The _________________ is the interest on the installment loan:

Finance charge = Total installment price - Cash price.

Example 1: The cost of a used pick-up truck is \$9345. We can finance the truck by paying \$300 down and \$194.38 per month for 60 months.

a. Determine the amount financed.

b. Determine the total installment price.

c. Determine the finance charge.

■ **Open-end Installment Loans**

- Using a credit card is an example of an open-end installment loan.
- Customers receive a statement each month.

■**Methods for Calculating Interest on Credit Cards:**

Use $I = Prt$, where r is the monthly rate and t is **one month.**

Unpaid balance method: The principal, P, is the balance on the first day of the billing period less payments and credits.

Example 1: Christian's credit card company starts each billing period on the first day of each month, and it uses the unpaid balance method. On the last day of January, Christian put airline tickets on his credit card, totaling $4000. His credit card charges 1.35% interest each month. Christian puts no other charges on his credit card for the rest of the year.

a) What is Christian's unpaid balance for February?

b) How much interest will Christian be charged during the month of February?

c) What is Christian's balance on the credit card by the end of February?

d) The credit card requires a $85 minimum payment. What is Christian's unpaid balance for the start of March, if he pays the minimum amount?

e) How much interest will Christian be charged in March?

f) What is Christian's balance on the credit card by the end of March?

Example 2: Christian's credit card company starts each billing period on the first day of each month, and it uses the previous balance method to calculate interest. His balance the last day of December was $5000.00. The credit card company charges 18% interest per year (so ________% per month.) The credit card company requires a minimum payment of $200 per month for Christian. Fill out the table for Christian's credit card.

Month	Amount of interest due	New balance with interest included	Ending balance with payment made.
January			
February			
March			
April			
May			
June			
July			
August			
September			
October			
November			
December			

Section 3.5: The Cost of Home Ownership

Mortgages

- A ***mortgage*** is a long-term loan for the purpose of buying a home.
- The down payment is the portion of the sale price of the home that the buyer initially pays to the seller.
- The ***amount of the mortgage*** is the difference between the sale price and the down payment.
- mortgages have the same monthly payment during the entire time of the loan.

Computations Involved with Buying a Home

- A document, called the ***Truth-in-Lending Disclosure Statement***, shows the buyer the APR for the mortgage.
- In addition, lending institutions can require that part of the monthly payment be deposited into an escrow account, an account used by the lender to pay real estate taxes and insurance.

Loan Payment Formula for Installment Loans

The regular payment amount, *PMT*, required to repay a loan of *P* dollars paid *n* times per years over *t* years at an annual rate *r* is given by

$$\text{Payment} = \frac{P\left(\frac{r}{n}\right)}{\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]}$$

Example 1: The price of a home is $195,000. The bank requires a 10% down payment since the buyer is a first-time home buyer. The cost of the home is financed with a 30-year fixed rate mortgage at 7.5%.

a. Find the required down payment.

$195000(.1) = \$19,500$

b. Find the amount of the mortgage. loan amt

$195000 - 19500 = \$175,500$

c. Find the monthly payment (excluding escrowed taxes and insurance).

$$\frac{175,500\left(\frac{.075}{12}\right)}{\left[1-\left(1+\frac{.075}{12}\right)^{-360}\right]} = \frac{1096.875}{.8938601707} = \$1227.12\cancel{1463}$$

$1227.12

d. Find the total amount paid by the owner over 30 years.

$(1227.12)(12)(30) = \$441,763.20$

Total in payments

Total w/down payment

$441763.20 + 19500 = \$461,263.20$

e. Find the total interest paid over 30 years.

Total payments − amt of loan

$441,763.20 - 175,500$

$\$266263.20$

amt interest paid

Example 2: The price of a home is \$465,000. The bank requires a 20% down payment at the time of closing. The cost of the home is financed with a 30-year fixed rate mortgage at 5.5%.

a. Find the required down payment.

$465000(.2) = 93{,}000$

b. Find the amount of the mortgage ← loan amt

465000 - 93000

= \$372,000

c. Find the monthly payment (excluding escrowed taxes and insurance).

$$\frac{372000\left(\frac{.055}{12}\right)}{1-\left[\left(1+\frac{.055}{12}\right)^{-(12)(30)}\right]} = \frac{1705}{.8072247477} = \$2112.18$$

d. Find the total amount paid by the owner over 30 years.

$(2112.18)(12)(30) = \$760{,}384.80$

e. Find the total interest paid over 30 years.

$760{,}384.80 - 372000 = \$388{,}384.80$ ← Total Interest

f. As another option, the family decides to consider a 20-year mortgage, still at 5.5% and with a 20% down-payment. Find the monthly payment and the total interest paid over 20 years.

$$\frac{372000\left(\frac{.055}{12}\right)}{1-\left[\left(1+\frac{.055}{12}\right)^{-12(20)}\right]} = \frac{1705}{.6662913069} = \$2558.94$$

raised payment by \$446.76

$2558.94(12)(20) = \$614145.60$ ← Total payments

$- 372000$

$242{,}145.60$ ← Total interest

20 year loan

We pay \$146,239.20 less in in interest